Name: ________________________________

Note that the questions are not weighted equally. Budget your time accordingly and do not work too long on any one problem. If you feel that a question is ambiguous, give your reasons and state your assumptions.

Double Angle Identities:

\[ \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \]

\[ \cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \]

Law of Cosines:

\[ A^2 = B^2 + C^2 - 2BC \cos \alpha \]

where \( A, B, C \) are the sides of the triangle and \( \alpha \) is the angle opposite side \( A \).

Denavit-Hartenberg Matrix:

\[
A_i = \begin{bmatrix}
\cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\
\sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\
0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Determinant of 3x3 matrix:

\[
\det \begin{bmatrix}
c_1 & c_2 & c_3 \\
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3
\end{bmatrix} = (a_2b_3 - a_3b_2)c_1 + (a_3b_1 - a_1b_3)c_2 + (a_1b_2 - a_2b_1)c_3
\]