Reading: *Principles of Robot Motion* Chapters 4 and 7.
Potential field methods for path planning, Sampling-based planners

Problems:

1. **Is the potential field planner shown in Algorithm 4 of [Choset] provably correct?** If so, sketch the proof. If not, explain why not.

2. **Is the potential field planner shown in Algorithm 4 of [Choset] provably complete?** If so, sketch the proof. If not, explain why not.

3. **Verify Equation 4.1 of [Choset].**

4. **Verify Equation 4.5 of [Choset].**

5. **Consider the family of navigation functions given by**
   \[ \phi_k(q) = \frac{\gamma_k(q)}{\beta(q)} \]
   **in which** \( \gamma_k \) **is given by Equation 4.9 [Choset], and** \( \beta(q) \) **is given by Equation 4.8 [Choset].** Show mathematically that for large enough \( k \), \( \phi_k \) has a single minimum at \( q = q^* \) (i.e., do not merely reproduce the qualitative argument given in [Choset]). Hints: (i) Remember that the domain of \( \phi_k \) is bounded (by the circle of radius \( r_0 \)). (ii) It is not necessary to explicitly compute \( \nabla \beta \).

6. **Consider a two-link planar arm with link links \( a_1 \) and \( a_2 \). Let** \( q_{init} = (0, 0) \), **and** \( q_{goal} = (\pi/2, \pi/2) \). **Assume that the workspace control points are located at the origins of D-H frames 1 and 2 (i.e., at the end of links 1 and 2), and that the workspace potential is the parabolic well. Compute the configuration space force vector for** \( q = (0, 0) \).

7. **As we discussed in class, RPP attempts to solve the local minimum problem that faces potential fields planners. For PRM, we can prove probabilistic completeness by (a) building a “\( \rho \)-tube” around a feasible path, and showing that PRM fails to a path in this tube with probability that exponentially decreases to zero with the number of samples.**
   
   Is it possible to construct a similar proof to show that RPP is probabilistically complete for the simpler problem of escaping the attractive basin of a particular local minimum in the artificial potential field? In particular, suppose there exists a path \( \gamma \) that escapes this attractive basin by following the gradient of the potential field, and further, that any path within distance \( \rho \) of the path \( \gamma \) will also escape the attractive basin by following the gradient of the potential field. Can we use a PRM-style approach to prove that a random walk will find a path in this \( \rho \)-tube? If so, sketch the proof. If not, explain why not.

8. **Consider a rigid object that moves freely in a 3-dimensional unit cube in Euclidean space, and whose orientation is represented by a unit quaternion.** The configuration space of this object is represented by \( Q = Q \times [0, 1] \times [0, 1] \times [0, 1] \), **in which** \( Q \) **denotes the set of unit quaternions.** Describe a procedure for generating random samples from \( Q \).

9. **Consider a robot whose dynamics are specified by the differential equation** \( \dot{x} = f(x, u) \), **in which** \( x \in X \) **denotes the state of the robot, and** \( u \in U \) **is the input.** **Can the postprocessing method described in Section 7.1.2 (Fig. 7.7) of [Choset] be used to smooth a path for this system?** Justify your answer.

10. **Consider a polygon \( \mathcal{A} \) with configuration space \( Q = SE(2) \).** Define the function \( \rho : SE(2) \times SE(2) \rightarrow \mathbb{R} \) **given by**
    \[ \rho(q_1, q_2) = \max_{a \in \mathcal{A}} ||a(q_1) - a(q_2)|| \]
    **in which** \( a(q) \) **denotes the position in the plane of** \( a \in \mathcal{A} \) **when** \( \mathcal{A} \) **is at configuration** \( q \). **Show that** \( \rho \) **is a metric.**