
Reading: *Principles of Robot Motion* Chapters 4 and 7.

Potential field methods for path planning, Sampling-based planners

Problems:

1. Is the potential field planner shown in Algorithm 4 of [Choset] provably correct? If so, sketch the proof. If not, explain why not.

2. Is the potential field planner shown in Algorithm 4 of [Choset] provably complete? If so, sketch the proof. If not, explain why not.

3. Verify Equation 4.1 of [Choset].

4. Verify Equation 4.5 of [Choset].

5. Consider the family of navigation functions given by

$$\phi_k(q) = \frac{\gamma_k(q)}{\beta(q)}$$

in which γ_k is given by Equation 4.9 [Choset], and $\beta(q)$ is given by Equation 4.8 [Choset]. Show mathematically that for large enough k, ϕ_k has a single minimum at $q = q^*$ (i.e., do not merely reproduce the qualitative argument given in [Choset]). Hints: (i) Remember that the domain of ϕ_k is bounded (by the circle of radius r_0). (ii) It is not necessary to explicitly computer $\nabla\beta$.

6. Consider a two-link planar arm with link links a_1 and a_2. Let $q_{\text{init}} = (0,0)$, and $q_{\text{goal}} = (\pi/2, \pi/2)$. Assume that the workspace control points are located at the origins of D-H frames 1 and 2 (i.e., at the end of links 1 and 2), and that the workspace potential is the parabolic well. Compute the configuration space force vector for $q = (0,0)$.

7. As we discussed in class, RPP attempts to solve the local minimum problem that faces potential fields planners. For PRM, we can prove probabilistic completeness by (a) building a "ρ-tube" around a feasible path, and showing that PRM fails to a path in this tube with probability that exponentially decreases to zero with the number of samples.

Is it possible to construct a similar proof to show that RPP is probabilistically complete for the simpler problem of escaping the attractive basin of a particular local minimum in the artificial potential field? In particular, suppose there exists a path γ that escapes this attractive basin by following the gradient of the potential field, and further, that any path within distance ρ of the path γ will also escape the attractive basin by following the gradient of the potential field. Can we us a PRM-style approach to prove that a random walk will find a path in this ρ-tube? If so, sketch the proof. If not, explain why not.

8. Consider a rigid object that moves freely in a 3-dimensional unit cube in Euclidean space, and whose orientation is represented by a unit quaternion. The configuration space of this object is represented by $\mathcal{Q} = Q \times [0,1] \times [0,1] \times [0,1]$, in which Q denotes the set of unit quaternions. Describe a procedure for generating random samples from $\mathcal{Q}$.

9. Consider a robot whose dynamics are specified by the differential equation $\dot{x} = f(x,u)$, in which $x \in X$ denotes the state of the robot, and $u \in U$ is the input. Can the postprocessing method described in Section 7.1.2 (Fig. 7.7) of [Choset] be used to smooth a path for this system? Justify your answer.

10. Consider a polygon $\mathcal{A}$ with configuration space $\mathcal{Q} = SE(2)$. Define the function $\rho : SE(2) \times SE(2) \to \Re$ given by

$$\rho(q_1, q_2) = \max_{a \in \mathcal{A}} ||a(q_1) - a(q_2)||$$

in which $a(q)$ denotes the position in the plane of $a \in \mathcal{A}$ when $\mathcal{A}$ is at configuration q. Show that ρ is a metric.